Rotordynamic Analysis of a Small Rotor-Bearing System

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Abstract

Dynamic analysis of turbomachinery rotors is vital in the design and development stages of turbomachinery engines. The vibration problems that can be occurred in the operational conditions may cause damage to the rotating parts of the engine or even failure of the engine completely. For this purpose, effective approaches such as finite element method are used to model the complex shape of the rotor and analyse the dynamic behaviour of the rotor. In this study, full rotor dynamic analysis of a certain rotor-bearing system is carried out using a Timoshenko beam finite element model and a 3D finite element model. The lateral vibration behaviour of the rotor is predicted using 1D FE model and the coupled lateral-torsional dynamic characteristics of the rotor is obtained using the 3D FE model.

Keywords: Dynamic analysis; Campbell diagram; Unbalance response

1. Introduction

Rotating machines are one of the fundamental components of engineering systems and are widely used in the industry. These machines should be designed carefully in order to avoid catastrophic failures during the operation. Thus, it is important and necessary to determine the dynamic characteristics of rotors that are exposed to vibration for the design and maintenance of the machines.

Al-bedoor [1] presented a dynamic model for a typical elastic blade attached to a disk driven by a shaft which is flexible in torsion. He employed the Lagrangian approach in conjunction with the finite element method in deriving the equations of motion, within the assumption of small deformation theory. Yang and Huang [2] studied the effects of disk flexibility, blade's stagger angle and rotational speed upon the natural frequencies and mode shapes of a shaft-disk-blade system. They derived the equations of motion from the energy approach in conjunction with the assumed modes method. Yang and Huang [3] analyzed the dynamic behavior of a coupled shaft-disk-blade system. They found out that the flexibility of the disk evolves the blade-blade modes into disk-blade and blade-blade modes, and causes frequency loci veering and merging with rotation. Whalley and Abdul-Ameer [4] investigated the dynamics of shaft-rotor systems where the shaft profile are contoured and the shaft diameter is the function of the shaft length. They obtained the critical speeds of the system using simple harmonic response methods. Nevzat [5] adopted analytical method to explore the shaft-disk system and found the 1st and 2nd critical speeds theoretically and experimentally. Dynamic analysis of multi-stage cyclic structures was reported in [6-8].

Jalali et al. [9] predicted the dynamic behaviour of a rotor-bearing system with a 1D finite element model, a 3D finite element model and experimental modal test. They obtained natural frequencies and mode shapes of the rotor at rest under free-free boundary condition using beam model, 3D model and modal test. Also, they performed a full rotor dynamic analysis for the rotor by the use of both models.

Taplak and Parlak [10] analysed the dynamic behaviour of a gas turbine rotor using a program based on Timoshenko beam finite elements. They obtained critical speeds, Campbell diagram and operational deflection shapes of the rotor to completely predict the dynamic behaviour of the rotor in the operating conditions. Creci et al. [11] performed a full rotor dynamic analysis for a 5-KN thrust gas turbine taking into account the bearing dynamics. The shaft of the studied single spool gas turbine was supported by a deep groove ball bearing and a squeeze film damper. Yu et al. [12] proposed a finite element model using a 3-node spatial element based on Timoshenko beam theory which provided by ANSYS package for modal analysis of crankshaft. They obtained the natural frequencies and mode shapes of the crankshaft with the proposed model. Zeighampour et al. [13] used Dynrot program based on beam elements to analyse the dynamic behaviour of a four spool gas turbine rotor. They obtained the four first mode shapes of the rotor and the Campbell diagram.

In this study, 1D Timoshenko beam finite element model and 3D finite element model based on solid elements are used for modal analysis of a certain high speed gas turbine rotor in non-rotating conditions and to analyse the dynamic behaviour of the rotor in the operating conditions. The first bending mode shapes of the rotor in the non-rotating situation are depicted using 1D FE model. The Campbell diagram of the rotor is obtained using both models and the unbalance response of the rotor to center of mass imbalance at the compressor is calculated to validate the critical speed calculated from the Campbell diagrams.

2. Equations of motion

The equations describing the motion of even a simple rigid body with mass *m* and principal moments of inertia J_ξ , J_η, and J_ζ referred to a reference frame $\xi\eta\zeta$ fixed to it in the three dimensional space are actually complex, particularly when dealing with the rotational degrees of freedom, and they do not allow the direct use of any linear model. With reference to an inertial frame *xyz* and a rotating frame $\xi\eta\zeta$ fixed to the rigid body and coinciding with its principal axes of inertia, the six equations of motion under the action of the generic force $\vec{F}$ and moment $\vec{M}$ can be written in the form [14].

$$
\begin{aligned}
m\ddot{x} &= F_x \quad , \quad M_\xi = \dot{\Omega}_\xi J_\xi + \Omega_\zeta \Omega_\eta (J_\zeta - J_\eta) \\
m\ddot{y} &= F_y \quad , \quad M_\eta = \dot{\Omega}_\eta J_\eta + \Omega_\xi \Omega_\zeta (J_\xi - J_\zeta) \\
m\ddot{z} &= F_z \quad , \quad M_\zeta = \dot{\Omega}_\zeta J_\zeta + \Omega_\xi \Omega_\eta (J_\eta - J_\xi)
\end{aligned}
\tag{1}
$$

The three equations for the rotational degrees of freedom, which are the well-known Euler equations, are clearly nonlinear in the angular velocity $\vec{\Omega}$.

However, a number of simplifications allow a linearized model to be obtained that retains the basic features of the dynamic behaviour of rotating systems and allow us to describe it correctly, both in a qualitative and a quantitative manner.

The two assumptions of small unbalance and small displacements allow the linearization of the equations of motion in a way that is consistent with what is usually done in the dynamics of structures. However, even in the case of the discretised model of a linear rotor that is axially symmetrical about its spin axis and rotates at a constant spin speed Ω, the linearized equation of motion (dynamic equilibrium equation) is of the following general form[14]:

$$\mathbf{M}\ddot{\mathbf{q}}(\mathrm{t}) + (\mathbf{C} + \mathbf{G})\dot{\mathbf{q}}(\mathrm{t}) + (\mathbf{K} + \mathbf{H})\mathbf{q}(\mathrm{t}) = \mathbf{f}(\mathrm{t}) \tag{2}$$

where $\mathbf{q}(\mathrm{t})$ is a vector containing the generalized coordinates, referred to an inertial frame, $\mathbf{M}$ is the symmetric mass matrix, $\mathbf{C}$ is the symmetric damping matrix, $\mathbf{G}$ is the skew-symmetric gyroscopic matrix, $\mathbf{K}$ is the symmetric stiffness matrix, $\mathbf{H}$ is the skew-symmetric circulatory matrix, and $\mathbf{f}(\mathrm{t})$ is a time-dependent vector in which all forcing functions are listed.

When dealing with rotating systems, one of the forcing functions is usually that caused by the residual unbalance that, although small, cannot nevertheless be neglected. Unbalance forces are harmonic functions of time, with an amplitude proportional to Ω^2 and a frequency equal to Ω.

The gyroscopic matrix contains inertial, and hence conservative terms that, in the case of rotor dynamics, are strictly linked with the gyroscopic moments acting on the rotating parts of the machine. If the equation is written with reference to a non-inertial frame, terms linked with Coriolis acceleration also are present in the gyroscopic matrix. The circulatory matrix contains non-conservative terms linked with the internal damping of rotating elements and, when using a linearized model for fluid bearings or seals, with the damping of the fluid film surrounding the rotor. In this paper, the circulatory matrix is equal to zero because damping is neglected in the models.

Equation (2) is that of a non-natural, circulatory system and hence differs from the typical equations encountered in dynamics of structures, where all matrices are symmetric. It must be noted that in rotor dynamics, the gyroscopic and circulatory matrices $\mathbf{G}$ and $\mathbf{H}$ are proportional to the spin speed Ω, and when Ω tends to zero, the skew-symmetric terms vanish and the equation reduces to that of a still structure. Also, if $(\mathrm{t}) = 0$ as well as $\Omega = 0$, the equation (2) can be used for modal analysis of the system in non-rotating situation. In addition, the damping and stiffness matrices $\mathbf{C}$ and $\mathbf{K}$ may depend on the spin speed, often on its square Ω^2, and $\mathbf{H}$ can be a more complex function of Ω.

Most flexible rotors can be considered as beam-like structures. Under fairly wide assumptions, the lateral behavior of a beam can be considered as uncoupled from its axial and torsional behavior. The same uncoupling is usually assumed in rotor dynamics, with the difference that no further uncoupling between bending in the principal planes is possible. When the flexural behavior can be uncoupled from the axial and torsional ones, Equation (2) holds for the first one, and the torsional and axial equations of motion are usually those of a natural, non-circulatory system [14].

In this paper, the matrices of the beam and solid elements should be calculated from the formulation of the finite element method and the matrices in the equation (2) should be obtained by assembling the global matrices for the whole finite element model [10-14]. When the natural frequencies of the rotor at various rotor speeds are calculated, the Campbell diagram can be plotted. The natural frequencies of the rotor at various rotor speeds can be calculated by solving the eigenvalue problem form of the equation (2). In addition, The unbalance response of the rotor can be obtained by obtaining the solution of the equation (2) when $\mathbf{f}(\mathrm{t})$ is a harmonic function of time, with an amplitude proportional to Ω^2 and a frequency equal to Ω.

3. Finite element models

3.1 Beam finite element model

The shaft of the studied rotor-bearing system is made of aluminum alloy, the turbine is made of steel and the impeller is made of Titanium alloy. Fig. 1 shows the model of the rotor with Timoshenko beam elements. The model consists of 22 Timoshenko beam elements. The turbine disk is modelled using two large beams with the density of zero. A concentrated mass is used to model the turbine's inertial properties. The inertial properties of the turbine are exactly equal to those modelled in the 3D finite element model. The compressor disk is modelled using two massless beams and a concentrated mass with inertial properties equals to those modelled in the 3D finite element model.

Two springs are used to model the bearings. Every node used in the system has 4 degrees of freedom. These include translations in the nodal directions and rotations about nodal axes. Table 1 presents the mechanical and geometric properties of the elements and Table 2 indicates the characteristics of the concentrated masses.

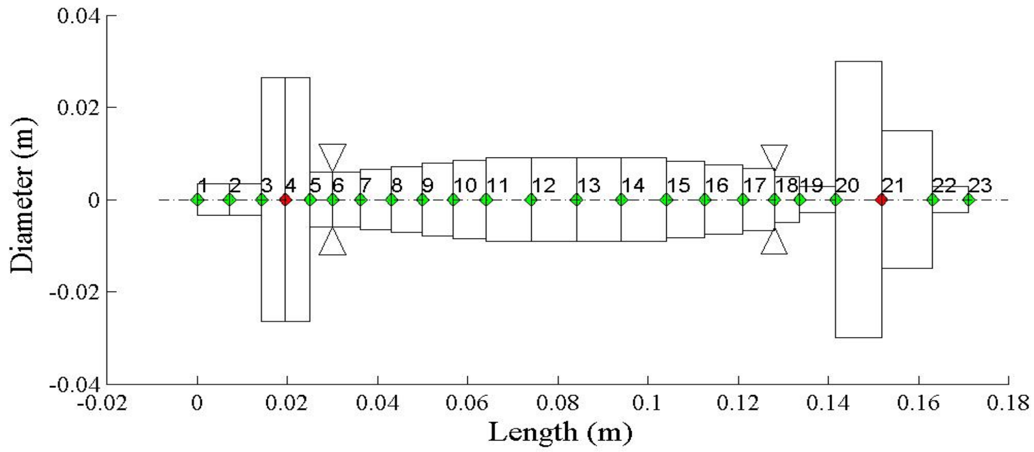

Figure 1. Beam finite element model of the rotor

Table 1. Mechanical and geometric properties of the elements

Element Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
d_o (mm)	7	7	53	53	12	12	13.2	14.4	15.8	17	18
l (mm)	7.1	7.1	5.3	5.4	5	6.1	6.8	6.9	6.9	7.2	10
ρ ($^{kg}/_{m^3}$)	2770	2770	0	0	2770	2770	2770	2770	2770	2770	2770
E (GPa)	72	72	200	200	72	72	72	72	72	72	72
Element Number	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
d_o (mm)	18	18	18	16.5	15	13.6	10	5.5	60	30	6.5
l (mm)	10	10	10	8.4	8.4	6.9	5.7	8	10.1	11.3	8
ρ ($^{kg}/_{m^3}$)	2770	2770	2770	2770	2770	2770	2770	2770	0	0	2770
E (GPa)	72	72	72	72	72	72	72	72	96	96	72

Table 2. Characteristics of concentrated masses

Mass number	1	2
Node number	4	21
m (g)	74.79	84.42
J_p (kgm^2)	2.61×10^{-5}	1.91×10^{-5}
J_d (kgm^2)	1.32×10^{-5}	1.28×10^{-5}

3.2 3D finite element model

The coupled lateral-torsional vibration analysis of the rotor is carried out using 3D FEM. The Ansys software is used to construct the 3D finite element model with solid elements. Figure 2 shows the 3D finite element model of the rotor. The model consists of 61557 nodes and 31365 solid elements. The material and inertial properties of this rotor model are equal to that of the beam finite element model. Two spring elements in the two lateral directions are used in place of the bearings to model the bearings.

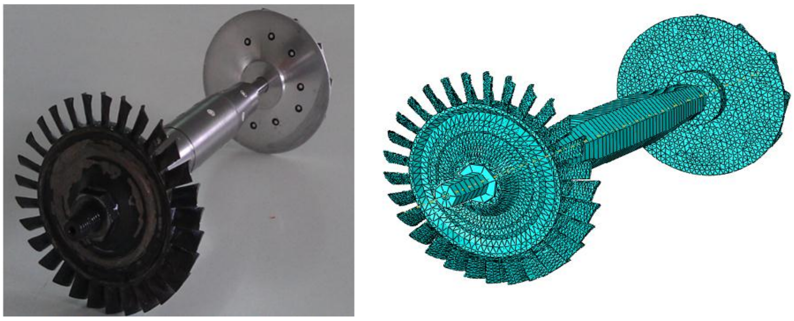

Figure 2. 3D finite element model of the rotor

4. Results

4.1 Modal analysis of the non-rotating rotor

The first 2 bending modes of the rotor at rest obtained by the beam model are depicted in Figures 3-4. It should be noted that, because of the axial symmetry of the rotor, the bending modes in two lateral directions are identical. Thus, in Figures 3-4, only one of the modes of two directions are showed.

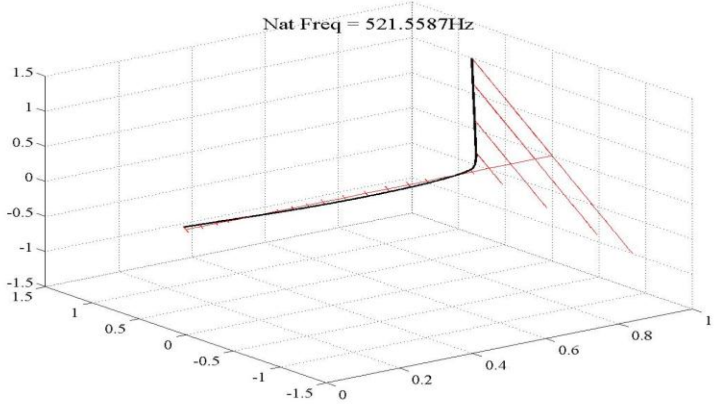

Figure 3. The first bending mode shape

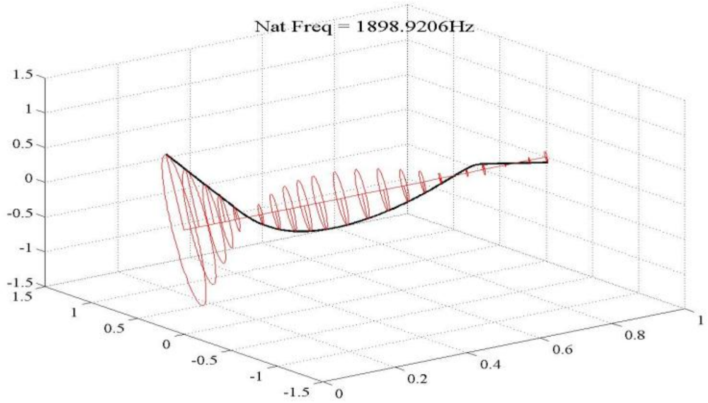

Figure 4. The second bending mode shape

The bending natural frequencies of the rotor obtained from two FE models are presented in Table 3. It is obvious that good agreement exists between the results.

Table 3. Bending natural frequencies

Number of bending mode	Beam FE model (Hz)	3D FE model (Hz)	Error (%)
1	521.55	528.69	1.3
2	1898.92	1953.60	2.7

The first torsional mode of the rotor obtained from 3D finite element model is showed in Figure 5. This mode is between the first and second bending modes. This mode cannot be obtained by the beam FE model because the torsional degrees of freedom are neglected in the formulation. The natural frequency of this torsional mode is 681.1 Hz.

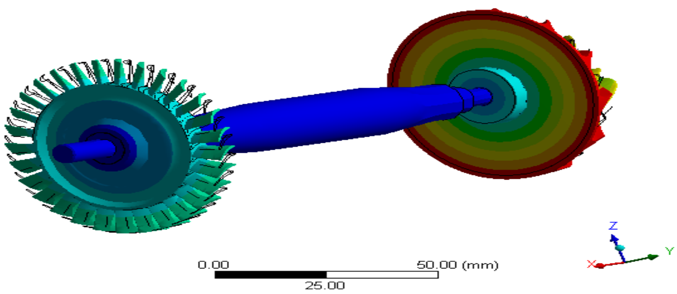

Figure 5. The first torsional mode shape

4.2 Dynamic analysis of the rotating rotor

The dynamic behaviour of the rotor in operating condition can be evaluated by obtaining the unbalance response of the rotor to the center of mass imbalance at the compressor or turbine. The speed in which response of the rotor peaks, is the critical speed because the natural frequency of the rotor coincides with the excitation frequency. In the Campbell diagram, the critical speed of the rotor is the speed corresponding to the intersection of the $\omega = \Omega$ line with the forward whirl curve.

An imbalance of 0.08442×10^{-6} kg.m at the gravity center of the compressor is considered in the beam FE model. Figure 6 shows the unbalance response evaluated at nodes 20, 21.

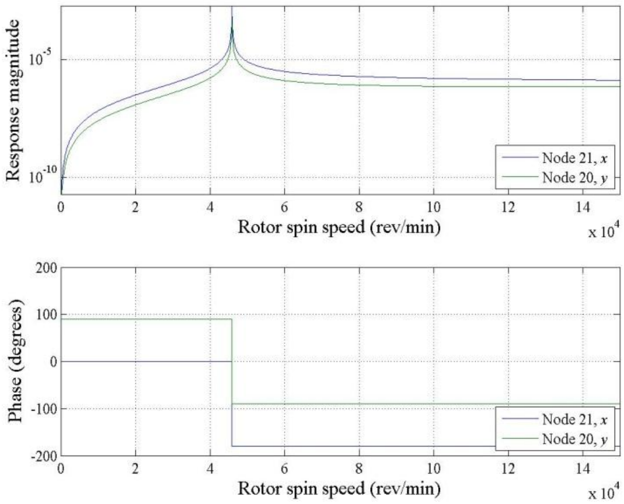

Figure 6. Unbalance response (Beam Model)

Figures 7-8 show the Campbell diagrams obtained from beam FE model and 3D FE model, respectively. The numerical analyses are performed considering speeds ranging from 0 to 150000 rpm and the damping is neglected in analyses.

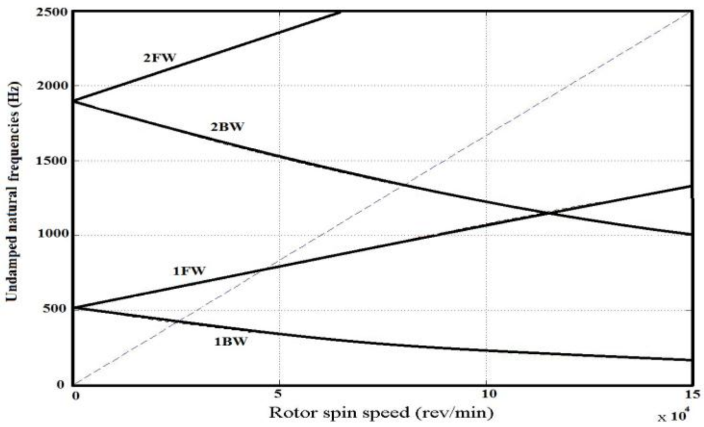

Figure 7. The Campbell diagram (beam FEM)

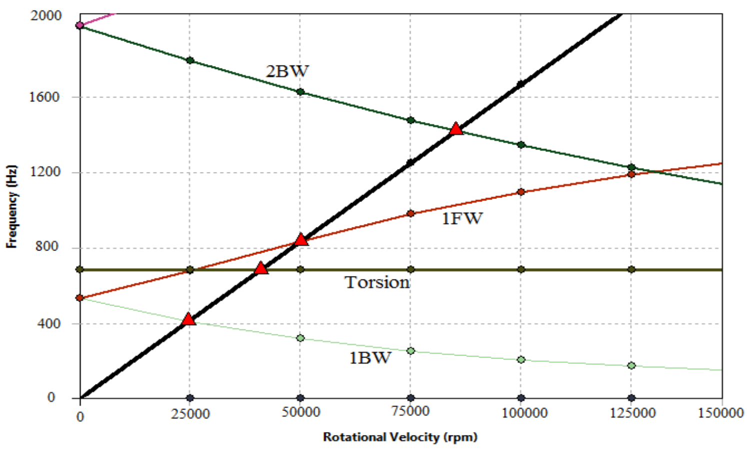

Figure 8. The Campbell diagram (3D FEM)

The bending critical speeds obtained from both the three-dimensional and beam models are presented in Table 4. As can be seen, the bending critical speed obtained from the Campbell diagrams are in good agreement with the speed in which the response peaks in Figure 6. Also, the operational deflection shapes of the rotor at 1B, 1F and 2B which are showed in [9] proves the correctness of the critical speed.

Table 4. First bending critical speed

	Beam model (rpm)	3D FE model (rpm)	Δ (%)
Bending critical speed	45894	50142	8.47

5. Conclusions

The coupled and uncoupled vibration analysis of a high speed rotor-bearing system is carried out using a beam finite element model and a 3D finite element model. Modal analysis of the rotor at rest is carried out by the use of both models and good agreement exists between the results. In addition, the dynamic behaviour of the rotating system is investigated by obtaining the Campbell diagram and critical speeds by the use of both beam and 3D FE model and the unbalance response by the use of beam model. From numerical analyses, it is found that the rotor cannot be in the resonance situation because the critical speed of the rotor is far from the operating speed range of the rotor.

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