



# COMPLEX MODES

Modal Analysis and Modal testing

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# Real Modes

**Complex eigenvalues:**

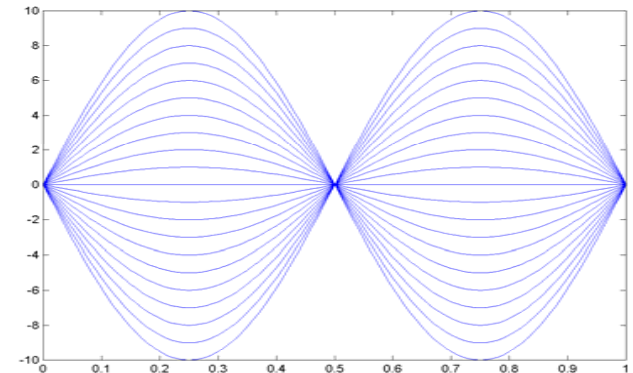
Decay

Oscillatory

$$\lambda = a + ib$$

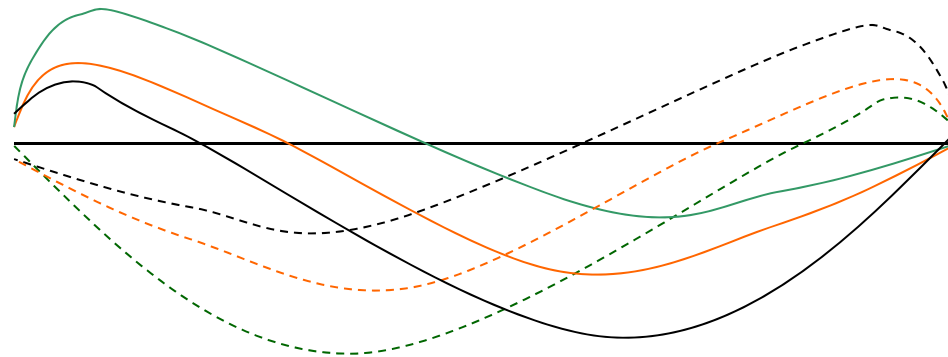
**Real modes:**

- The phase angle is 0 or 180
- All parts of the structure reach their own maxima at the same instant in the vibration cycle.
- All parts of the structure pass through their zero deflection position at the same time.
- There are two moments in each vibration cycle when structure is completely undeformed.

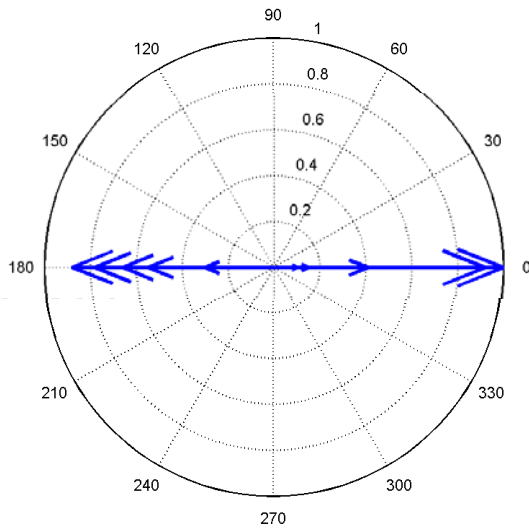


# Complex Modes

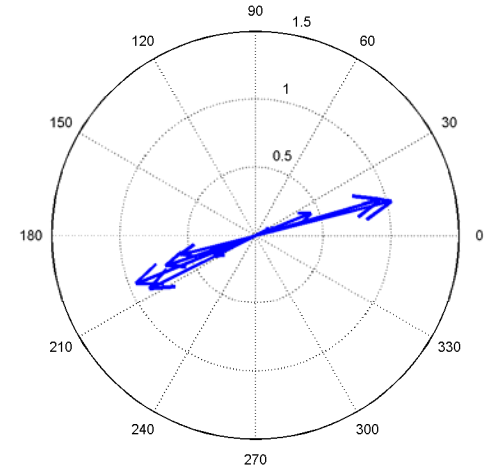
- ✦ Different points reach their maxima at different times.
- ✦ Different points reach their zero position at different times.
- ✦ While the real mode has the appearances of standing wave, the complex mode is better described as exhibiting traveling wave.



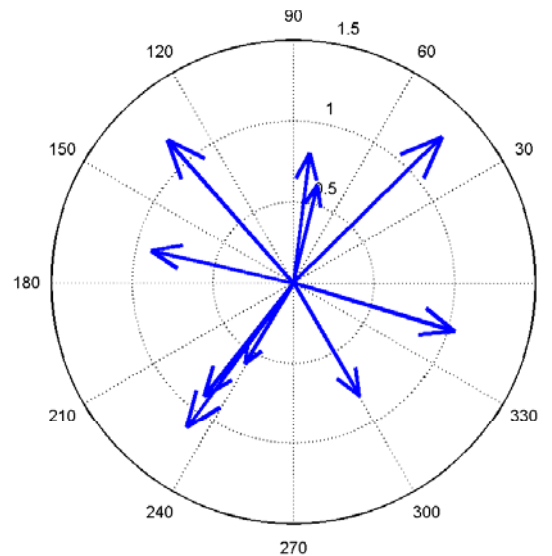
# Display of Complex Mode



Real mode



Complex mode



Complex mode

# Measurement of Modal Complexity

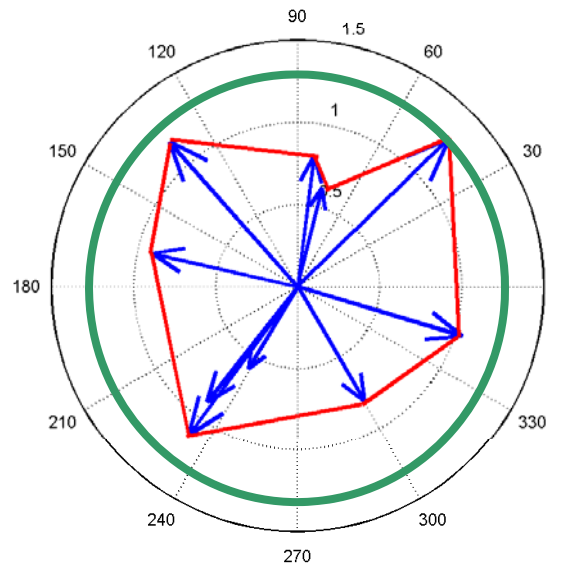
- ✦ The aim is to find the degree of complexity of a particular mode.
- ✦ Two will be discussed here.
- ✦ There is not any universally-accepted indicator.
- ✦ MCF1 simply measures the phase difference between all pairs of mode shape vector elements, regardless of the magnitude of those elements.

$$MCF1 = \sum_{j=1}^N \sum_{k=1, k \neq j}^N (\theta_{rj} - \theta_{rk})$$

# Measurement of Modal Complexity

MCF2 reflects the magnitude as well as the phase of Each of the elements.

$$MCF2 = \frac{\text{Area of the polygon}}{\text{Area of the circle based on the length of the largest vector element}}$$





# Origin of Complex Modes

- ✦ Complex modes can exist in structures which contain rotating components.
- ✦ In linear structures the mode shapes can be complex only if the damping is distributed in a non-proportional way.
- ✦ The hysteretic damping of most structural elements is distributed proportionally.
- ✦ The majority of the damping in real structures usually concentrated at joints which result in a nonproportional distribution.





# Origin of Complex Modes

- ✦ It is found that non-proportionality is a necessary condition for complex modes to exist. However, it is not sufficient.
- ✦ It is found that close modes can create significant complexity in a structure. Again, this is a necessary condition not sufficient.
- ✦ Structures with repeated roots.